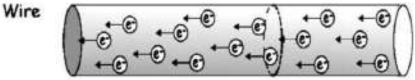
UNIT-3

CURRENT ELECTRICITY

Electric Current

The time rate of flow of charge through any cross-section of a conductor is called electric current.

$$\boxed{\text{Electric Current } I = \frac{q}{t}}$$



If the rate of flow of charge varies with time, then current at any time (instantaneous current) is given by -

$$\boxed{I = \frac{dq}{dt}}$$

- SI Unit of the current is **Ampere (A)**.

1 Ampere -

The current through a wire is said to be 1 ampere, if one coulomb of charge is flowing per second through any cross section of wire.

$$1 \text{ Ampere (A)} = \frac{1 \text{ Coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ C/s}$$

Current is a scalar quantity.

Though the electric current represents the direction of flow of positive charge, yet it treated as scalar, because current follows the law of scalar addition and not the law of vector addition.

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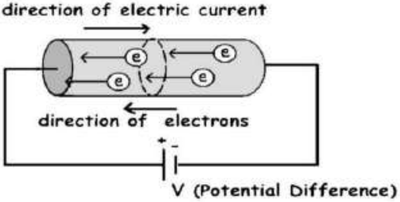
Direction of Electric Current

(i) Conventional Current

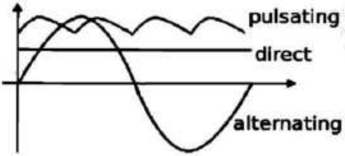
The direction of flow of positive charge gives the direction of current. It is called conventional current.

(ii) Electronic Current

The direction of flow of electrons gives the direction of electronic current. The direction of electronic current is opposite to the conventional current.



Different types of Current



Charge Carriers

Solids

(i) Conductors
Free electrons

(ii) Semiconductor
Free electrons and holes

(iii) Insulators
No charge Carriers

Liquids

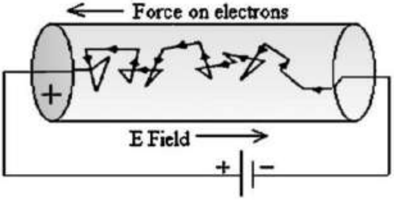
Positive and Negative Ions

Gas

Positive Ions and free electrons (At high Voltage)

Drift Velocity

It is defined as the average velocity with which the free electrons get drifted towards the positive end of the conductor under the influence of an external electric field applied.



Let E is the strength of applied electric field then the force on electrons due to applied field is -

$$F = eE \qquad (e = \text{charge on electron})$$

If m is the mass of electron, then acceleration Produced is given by $a = \frac{eE}{m}$

If τ is the relaxation time, which is average time that an electron spends between two Collisions, then

$$a = \frac{V_d - 0}{\tau} \quad \text{or} \quad V_d = a\tau$$

then $$\boxed{V_d = \left(\frac{eE}{m}\right)\tau}$$

* The average relaxation time (τ) is of the order of 10^{-14} second.
* Average Relaxation Time = $\dfrac{\text{mean free path of electron}}{\text{drift speed of electron}}$

Mobility (μ)

Mobility of charge carriers, responsible for the current is defined as the magnitude of drift velocity of charge per unit electric field applied.

$$\boxed{\mu = \frac{\text{drift Velocity}}{\text{Electric Field}} = \frac{V_d}{E}}$$

or $$\mu = \frac{qE\tau/m}{E} = \frac{q\tau}{m}$$

Mobility of electrons $\boxed{\mu_e = \dfrac{e\tau_e}{m_e}}$

Mobility of holes $\boxed{\mu_h = \dfrac{e\tau_h}{m_h}}$

* SI Unit of mobility is m^2/Vs.

The mobility of electrons is greater than holes. because the electrons are free to move anywhere inside the conductor while holes move from bond to bond.

$$\boxed{\mu_e > \mu_h}$$

Q. Explain how electron mobility changes for a good conductor when (a) the temperature of the conductor is decreased at constant potential difference and (b) applied potential difference is doubled at constant temperature.

RBSE 2006

Sol.

The electron mobility $\boxed{\mu_e = \frac{e\tau}{m}}$

(a) μ_e increases with the decrease of temperature

(b) μ_e does not change with the increase of potential difference.

Q. A potential difference of 12 V is applied across a conductor of length 0.2 m. If the drift velocity of electron is 3×10^{-4} m/s then calculate the electron mobility.

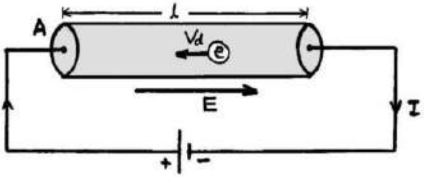
Sol. We know $E = V/l$

so $E = 12/0.2 = 60$ V/m.

and $\mu = \frac{V_d}{E} = \frac{3 \times 10^{-4}}{60} = 5 \times 10^{-6}$ m²/Vs

Relation between drift velocity and Current

Consider a Conducting wire of length L and having uniform cross section area A in which electric field is present



Let there are n free electrons per unit volume moving with the drift velocity V_d.

Total Volume of the conductor $= A \times l$

Total Number of free electrons $= n \times Al$

Total charge on free electrons $= -nAle$

So that Current $I = \frac{Q}{t}$

$$I = -\frac{nAle}{t}$$

$$\boxed{I = -neAV_d} \qquad \left(\because V_d = \frac{l}{t}\right)$$

Q. The drift velocity of electrons in a conductor is 10^{-4} m/s yet a bulb glows instantly when switched on, why?

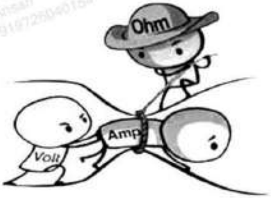
Answer – When we close the circuit, the electric field is setup in the entire closed circuit instantly with the speed of electromagnetic waves which causes electron drift at every portion of the circuit. Due to it, the current is setup in the entire circuit instantly.

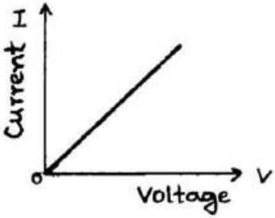
Ohm's Law -

The current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor, if the physical state of the conductor (Temperature, mechanical strain etc.) remains unchanged.

Mathematically $V \propto I$ or $\boxed{V = IR}$

R = resistance





* Value of resistance R depends upon the nature, dimensions of the conductor.
* SI Unit of resistance is ohm (Ω).

Resistance (R)

The resistance of conductor is the opposition offered by a conductor to the flow of electri current through it.

$$1 \text{ ohm} = \frac{1 \text{ Volt}}{1 \text{ Ampere}} = \frac{1V}{1A}$$

Thus 1 ohm is the electrical resistance of a conductor through which a current of 1 ampere flows when a potential difference of 1 volt is applied across the ends of the conductor.

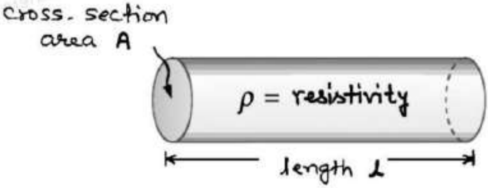
* Dimension of resistance is $[M^1L^2T^{-3}A^{-2}]$

Factors on which Resistance depends

(1) Length of Conductor -

Resistance of a conductor is directly proportional to the length of conductor.

$$R \propto l$$



(2) Area of Cross - Section of Conductor -

Resistance of a conductor is inversely proportional to the area of cross. section of conductor.

$$R \propto 1/A$$

(3) Nature of material & Temperature -

From above, $R \propto \frac{l}{A}$

Or $$\boxed{R = \frac{\rho l}{A}}$$

where ρ is known as specific resistance or electrical resistivity of the material of the conductor.

Electrical Resistivity (ρ) -

If $l = 1\,m$, $A = 1\,m^2$ then

$$R = \rho$$

Thus, the resistivity of the material of a conductor is defined as the resistance of a unit length with unit area of cross section of the material of the conductor.

* SI Unit of electrical resistivity is **ohm - meter** ($\Omega \cdot m$)

factors affecting Electrical Resistivity

By ohms Law, $V = IR$

so $$V = neAV_d R = neA\left(\frac{eE\tau}{m}\right)R$$

$$V = Rne^2A\left(\frac{V}{l}\right)\frac{\tau}{m} \qquad \left(\because E = \frac{V}{l}\right)$$

or $$R = \frac{m}{ne^2\tau} \times \frac{l}{A} = \frac{\rho l}{A}$$

so $$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Thus resistivity of Conductor depends upon -

(i) $\rho \propto 1/n$ Since the value of n depends upon the nature of material, hence the resistivity of the conductor depends upon the nature of material.

(ii) $\rho \propto 1/\tau$. With the increase in temperature of metal conductor, τ decreases hence its ρ increases.

* $\rho \propto 1/n$, So that If $n\uparrow$ then $\rho\downarrow$
* $\rho \propto 1/\tau$, So that If $T\uparrow$ then $\tau\downarrow$ so $\rho\uparrow$

Q. (CBSE 2016) Two wires one of copper and the other of mangnin wires having same resistence and equal thickness which wire is longer?

Sol. Resistance of Copper wire $R_{cu} = \frac{\rho_{cu} l_{cu}}{A}$

Resistance of mangnin wire $R_{mang.} = \frac{\rho_{mang.} l_{mang.}}{A}$

According to the question $R_{cu} = R_{mang.}$

$$\rho_{cu} l_{cu} = \rho_{mang} l_{mang.}$$

Since $\rho_{cu} << \rho_{mang.}$

So that $l_{cu} >> l_{mang.}$

i.e. Copper wire would be longer.

Q. (CBSE 2016, 19) When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is $8 \times 10^{28}\ m^{-3}$, calculate the resistivity of the material of wire.

Sol. We know $I = neAV_d$

Also $I = \frac{V}{R}$ and $R = \frac{\rho l}{A}$

So that $\frac{V}{R} = neAV_d$

$$\frac{V}{neV_d l} = \frac{RA}{l}$$

or $\rho = \frac{V}{neV_d l}$

$$\rho = \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$$\rho = 1.56 \times 10^{-5}\ \Omega m$$

Variation of Resistivity / Resistance with Temperature

Resistivity and resistance of the metallic conductor increases with increase on temperature.

The resistance of a conductor is directly proportional to the change in temperature and initial resistance so that

$$R_2 - R_1 \propto R_1 \quad \text{and} \quad R_2 - R_1 \propto (t_2 - t_1)$$

$$R_2 - R_1 \propto R_1(t_2 - t_1) \quad \text{or} \quad R_2 - R_1 = \alpha R_1 (t_2 - t_1)$$

Hence temperature variation of the resistance can be given as-

$$\boxed{R_2 = R_1 [1 + \alpha(t_2 - t_1)]}$$

or

$$\boxed{\alpha = \frac{R_2 - R_1}{R_1 \times (t_2 - t_1)}}$$

Thus temperature coefficient of resistance is defined as the increase in resistance per unit original resistance per degree celsius or Kelvin rise of temperature.

We know that $R = \rho l / A$

For small temperature variation, resistivity of the most of the metal varies according to the given relation -

$$\boxed{\rho_2 = \rho_1 [1 + \alpha(t_2 - t_1)]}$$

Note -

(i) For metals α is positive, therefore resistance of a metal increases with rise in temperature.

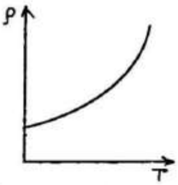
(ii) For insulator and semiconductors α is negative, so the resistance decreases with rise in temperature.

(iii) Resistivity of an electrolyte Solution -

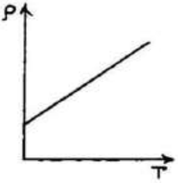
By increasing the temperature of electrolyte solution its resistance decreases. Because with increase in temperature the viscosity decreases, due to which the ions in it can move more freely in it. Hence the resistivity of electrolyte solution decreases.

★ The temperature coefficient of resistivity is negative for electrolyte solution.

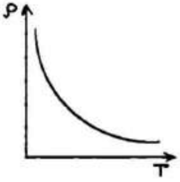
Effect of temperature on Resistance and Resistivity



(i) For Metals



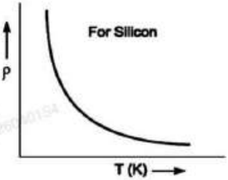
(ii) For Mangnin or Constentan

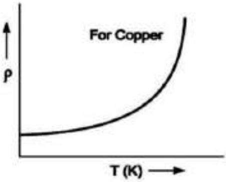


(iii) For semiconductor

Q. (CBSE 2013) Two materials Si and Cu, are cooled from 300K to 60 K. What will be the effect on their resistivity?

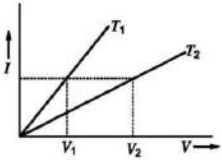
Sol. In Silicon, the resistivity increases.
In Copper, the resistivity decreases.





Q. (CBSE 2015, 21) I-V graph for a metallic wire at two different temperatures T_1 and T_2 is as shown in fig. Which of the two temperatures is lower and why?

Sol. If a constant current I flows through the conductor, resistance at temperature T_1 and T_2 is



$$R_1 = \frac{V_1}{I}$$

and $$R_2 = \frac{V_2}{I}$$

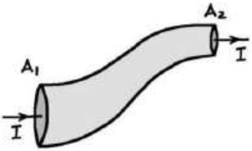
Since $V_2 > V_1 \Rightarrow R_2 > R_1$

The resistance of the wire increases with rise of temperature, Hence T_1 is lower than T_2.

Current Density (J)

current density (J) at a point in a conductor is defined as the amount of current flowing per unit area of the conductor around that point provided the area is held in a direction normal to the current.

$$\boxed{J = \frac{I}{A}}$$



as $I = neAV_d$

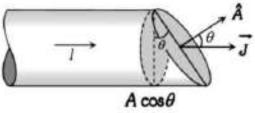
$$\boxed{J = \frac{I}{A} = neV_d}$$

The unit of current density is **ampere/metre²** [A/m^2]

Note- If the area vector of the conductor is not along the current then to calculate current density we should take the component of area vector along the current.

$$J = \frac{I}{A\cos\theta}$$

or $$I = \vec{J}\cdot\vec{A}$$



Conductivity (σ)

The inverse of resistivity (ρ) of a conductor is called its electrical conductivity.

Conductivity $\boxed{\sigma = \frac{1}{\rho}}$

The unit of electrical conductivity is mho/meter.

Conductance (G)

The inverse of resistance (R) is called Conductance of a conductor.

Conductance $\boxed{G = \frac{1}{R}}$

The unit of conductance is mho ($\mho$)

Relation between J, σ and E

We know that $I = neAV_d$

$$I = neA\left(\frac{eE\tau}{m}\right) = \frac{ne^2 A\tau E}{m}$$

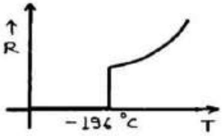
or
$$\frac{I}{A} = \frac{ne^2 \tau E}{m}$$

or
$$J = \frac{1}{\rho} E$$

$$\therefore \quad \boxed{J = \sigma E}$$

Superconductivity

In some metals and alloys when the material is cooled to a critical temperature (−196 °c) the resistance of the material falls to zero. This state of zero resistance is when the material become superconducting.



Use- Superconducting materials are used when very strong electromagnets are required, in MRI scanners or to reduce losses in power cables.

Super conductivity is used to produce very high speed computers.

Limitation of Ohm's Law / Non-ohmic Devices

Those devices which do not obey ohm's Law are called non-ohmic devices.

Examples - Vacuum tubes, semiconductor diode, transistor liquid electrolyte etc.

★ The semiconductor junction diode is also used to convert alternating current to direct current and are used to perform various logic functions.

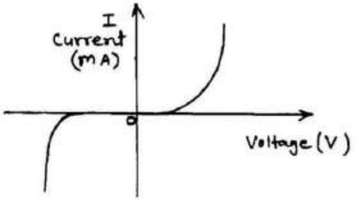
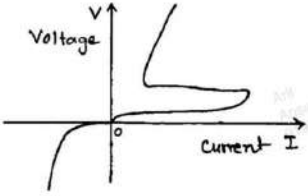


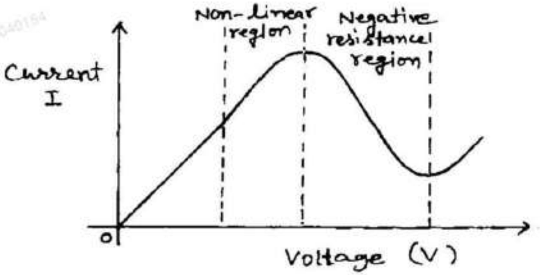
fig- Diode V-I Characteristics

★ The following graph shows the characteristics of a device known as a thyristor, which consist of four alternating layers of P-type and N-type semiconductors. Here the current carried by the device increases as the voltage decreases.



* A more fundamental breakdown of ohm's Law occurs in certain alloys for example GaAs at very Low temperatures (at 4K).

It is observed that the resistance of a uniform wire made of the alloy of length 2l is more than twice the resistance of the wire of length l. This can be understood in terms of the wave character of electrons, which leads to interference effects. This increases resistance.

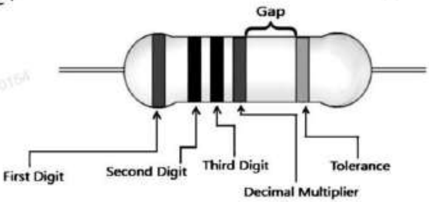


SHINE BRIGHT
LIKE A

Color Code for Carbon Resistors

Commercially resistors of different type and values are available in the market but in electronic circuits carbon resistors are more frequently used.

In carbon resistors value of resistance is indicated by four color bands marked on its surface as shown in figure.



Color	Figure(first and second band)	Multiplier(for third band)	tolerance
Black	0	1	-
Brown	1	10	-
Red	2	10^2	-
Orange	3	10^3	-
Yellow	4	10^4	-
Green	5	10^5	-
Blue	6	10^6	-
Violet	7	10^7	-
Gray	8	10^8	-
white	9	10^9	-
Gold	-	10^{-1}	5%
Silver	-	10^{-2}	10%
No Color	-	-	20%

The color of first and second band respectively gives the first and second significant figure of resistance and third band c gives the power of the ten by which two significant digits are multiplied, to obtain the value of the resistance.

Q. In a given resistor the first strip be red, second strip be brown and third strip be orange and fourth be gold then calculate the resistance of the resistor.

Sol.

$$21 \times 10^3 \pm 5\%$$

Q. The resistance of the given carbon resistor is $2.4 \times 10^6 \Omega \pm 5\%$. What is the sequence of colors on the strips provided on resistor?

Sol.

$$R = 2.4 \times 10^6 \Omega \pm 5\%$$

or

$$R = 24 \times 10^5 \Omega \pm 5\%.$$

The colors attached to the numbers 2, 4 and 5 are red, yellow and green. The color for 5% is gold.

Therefore, the colors of strips in sequence for the given carbon resistor are red, yellow, green and gold.

Q. The sequence of colored bands in two carbon resistor R_1 and R_2 is

(a) Brown, green, blue, Silver
(b) Orange, black, green

Find the ratio of their resistances.

Sol. According to color codes, resistance of two resistors are -

(a) $R_1 = 15 \times 10^6 \Omega \pm 10\%$

(b) $R_2 = 30 \times 10^5 \Omega \pm 20\%$

$\therefore$ Ratio of their resistances $\frac{R_1}{R_2} = \frac{15 \times 10^6}{30 \times 10^5}$

so $\frac{R_1}{R_2} = 5$

Q. CBSE 2008 Two metallic wires of the same material have the same length but cross sectional area is in the ratio 1 : 2. They are connected (a) in series and (b) in parallel. Compare the drift velocities of electrons in the two wires in both the cases.

Sol. Let each of two conductors is of length L.
Since, drift velocity of electron is given by -

$$V_d = \frac{eE\tau}{m} = \frac{e\tau V}{ml} \qquad \left(\because E = \frac{V}{l}\right)$$

$$V_d = \left(\frac{e\tau}{m}\right) \times \frac{V}{L}$$

At given temperature $\boxed{V_d \propto \frac{1}{l}}$ and potential difference.

(a) In series combination effective length $L_1 = 2l$

(b) In parallel Combination effective length $L_2 = l$

$$\therefore \quad \frac{V_{d_1}}{V_{d_2}} = \frac{l_2}{l_1} = \frac{l}{2l}$$

so $\quad V_{d_1} : V_{d_2} = 1 : 2$

Q. CBSE 2007 A voltage of 30 V is applied across a carbon resistor with first, second and third rings of blue, black and yellow colors respectively. What is the value of current in mA, through the resistor.

Sol. Resistance of Carbon resistor

$$R = 60 \times 10^4 \, \Omega \pm 20\ \%$$

$\because$ Current through the resistor $I = \frac{V}{R}$

$$I = \frac{30}{60 \times 10^4} = 0.5 \times 10^{-4} \text{ A}$$

or $\quad I = 0.05 \times 10^{-3} \text{ A} = 0.05 \text{ mA}.$

Q. (CBSE 2009) A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3 V battery. Find the current drawn from the battery.

Sol. Let original cross-sectional are and length of 15 Ω resistance are A and L and after stretching they become A' and L' respectively.

Initial resistance $R = \rho L/A = 15$

In case of stretching, volume of the wire remains same, so

$$AL = A'L'$$

so that $A' = \frac{A}{2}$ $\quad [L' = 2L]$

$\therefore$ Resistance after stretching

$$R' = \frac{\rho L'}{A'} = \rho\left(\frac{2L}{A/2}\right) = 4\left(\frac{\rho L}{A}\right)$$

so $R' = 4R = 4 \times 15$

Now resistance $R' = 60\ \Omega$

After dividing into parts, resistance of each part is 30 Ω.

$\therefore$ Effective resistance after connecting them into parallel combination is

$$R_{effective} = \frac{30}{2} = 15\ \Omega$$

Applied Potential diff., $V = 3\text{ V}$

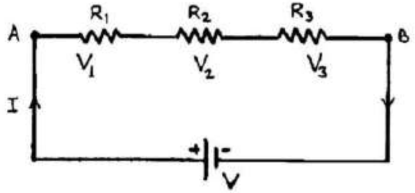
$\therefore$ Current drawn from the battery $I = \frac{V}{R_{effective}}$

$$I = \frac{3}{15} = \frac{1}{5}\text{ A}$$

Combination of Resistors

(A) Resistors in series

Resistors are said to be connected in series combination, if same current flows through each resistor when same potential difference is applied across the combination.



If V_1, V_2 and V_3 is the potential differene across each resistor R_1, R_2 and R_3 respectively due to the battery of potential difference V then

According to Ohm's Law -

$V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$

Since in series combination current remains same but potential is divided so.

$$\boxed{V = V_1 + V_2 + V_3}$$

or $\quad IR_s = IR_1 + IR_2 + IR_3$

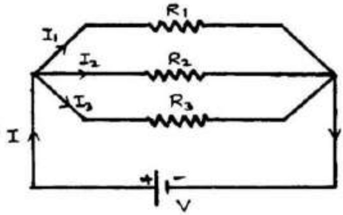
If R_s is the equivalent resistance of the series combination of R_1, R_2 and R_3 then -

$$V = IR_s$$

where $\boxed{R_s = R_1 + R_2 + R_3}$

(B) Resistors in parallel

Resistors are said to be connected in parallel combination if potential difference across each resistance is same.



Let a battery of potential difference V is connected across parallel combination of resistors R_1, R_2 and R_3.

Total current in the circuit is -

$$I = I_1 + I_2 + I_3$$

by applying Ohm's Law, we get -

$$V = I_1 R_1 = I_2 R_2 = I_3 R_3$$

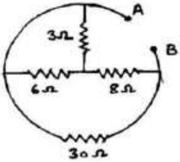
or $$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

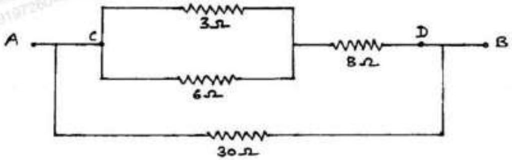
Hence for resistors connected in parallel combination, reciprocal of equivalent resistance is equal to the sum of reciprocal of individual resistances.

* The value of equivalent resistance for resistors connected in parallel combination is always less than the value of the smallest resistance in circuit.

Q. Find the equivalent resistance between A and B of the given circuit.



Sol. Equivalent circuit of the given circuit is -



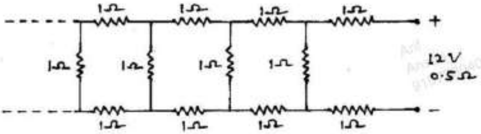
Equivalent resistance between C and D is

$$R_{CD} = \frac{3 \times 6}{3+6} + 8 = 10\ \Omega$$

Now in the given figure 10 Ω and 30 Ω are in parallel so equivalent resistance of the given circuit is -

$$R_{eq} = \frac{10 \times 30}{10 + 30} = 7.5\ \Omega$$

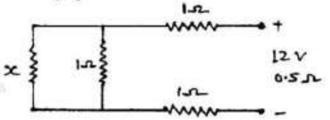
Q. Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the following infinite network; each resistor has 1 Ω resistance.



Sol. Let x be the equivalent resistance of infinite network.

Since the network is infinite, therefore the addition of one more unit of three resistances each of value 1 Ω across the terminals will not change the total resistance of network i.e. it should remain x.

Therefore the network would appear as shown in the given figure and its total resistance remain x.



Here the parallel combination of x and 1 Ω is in series with the two resistors of 1 Ω each.

The resistance of parallel combination is –

$$\frac{1}{R_P} = \frac{1}{x} + \frac{1}{1} = \frac{1+x}{x}$$

or $$R_P = \frac{x}{1+x}$$

Total resistance of the network will be given by.

$$x = 1+1+\frac{x}{x+1} = 2+\frac{x}{x+1}$$

$$x(x+1) = 2(x+1)+x$$

$$x^2+x = 2x+2+x$$

or $$x^2-2x-2=0$$

or $$x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2}$$

$$x = 1 \pm \sqrt{3}$$

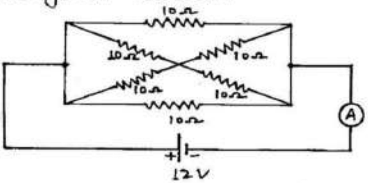
The value of resistance can not be negative, therefore the resistance of the network is.

$$x = 1+\sqrt{3} = 2.73\ \Omega$$

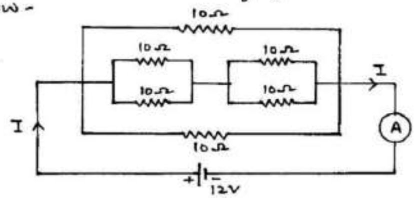
Total resistance of the network = 2.73 + 0.5 = 3.23 Ω

$\therefore$ Current drawn $I = \frac{12}{3.23} = 3.72\ A$

Q. Calculate the current shown by the ammeter A in the given circuit.



Sol. The equivalent circuit of given network is as given below -



The resistance of the middle arm is -

$$= \frac{10 \times 10}{10 + 10} + \frac{10 \times 10}{10 + 10}$$

$$= 5 + 5 = 10\ \Omega$$

So the Effective resistance of the circuit is -

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

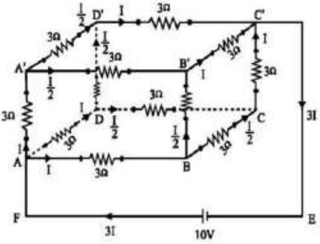
So $R = \frac{10}{3}\ \Omega$

Current shown by ammeter $I = \frac{V}{R}$

$$I = \frac{12}{10/3} = 3.6\ A$$

Q. CBSE 2019 Twelve wires each of resistance 3Ω are connected to form a cubical network. A battery of 10V and negligible internal resistance is connected across the diagonally opposite corners of this network. Find the equivalent resistance and the current along each edge of the cube.

Sol. Applying KVL to the loop ABCC'EFA



$$3I + 3\times\frac{I}{2} + 3I - 10 = 0$$

$$\frac{15I}{2} = 10$$

$$I = \frac{4}{3}\,A$$

$$R_{eq} = \frac{V}{3I} = \frac{10\times15}{3\times20}$$

$$R_{eq} = 2.5\,\Omega$$

$$\text{Current} = I_{AB}\,(= I_{AA'} = I_{AD} = I_{D'C'} = I_{B'C'} = I_{CC'}) = \frac{4}{3}\,A$$

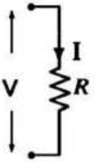
$$\text{and current} = I_{DD'}\,(= I_{A'B'} = I_{A'D'} = I_{DC} = I_{BC} = I_{BB'}) = \frac{2}{3}\,A$$

[redacted]

Electrical Power

The rate at which electric work is done by the source of emf in maintaining the current in electric circuit is called electric power of the circuit.

$$\text{Electrical Power} = \frac{\Delta U}{\Delta t} = \frac{V \times \Delta q}{\Delta t}$$



$$\boxed{P = V \times I = I^2 R = \frac{V^2}{R}}$$

Unit of electrical Power is 'Watt'.

1 Watt - The power of an electrical circuit is said to be one watt if one ampere current flows in it against a potential difference of 1 volt.

Electrical Energy

The total work done in maintaining the current in electric circuit by the source of emf is called electrical energy given to the circuit.

$$\text{Electrical Energy} = \text{Electrical Power} \times \text{time}$$

$$W = (V \times I) \times t$$

or $$W = I^2 R t = \frac{V^2}{R} t$$

If this work done appears as heat then amount of heat produced (H) is given by -

$$\boxed{H = W = I^2 R t = \frac{V^2}{R} t}$$

Above equation is Known as ***Joule's law of heating.***

Note -

1 Killowatt-hour or one unit is the electrical energy which is used in a circuit of power 1K watt for one hour.

$$1 \text{ Killowatt hour} = 1000 \text{ watt} \times 3600 \text{ sec}$$

$$\boxed{1 \text{ Killowatt hour (KWh)} = 3.6\times10^{6} \text{ Jule}}$$

* In mechanics the unit of power is 'horse power' (H.P.).

$$1 \text{ Horse Power (H.P.)} = 746 \text{ Watt}$$

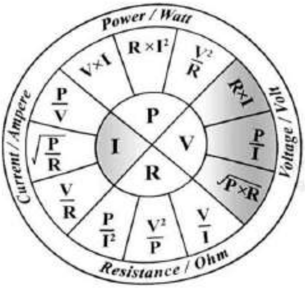
Number of Units -

If in an electrical circuit a current I is flowing at voltage V for time t hours then total electrical energy consumed in the circuit will be

$$W = P \times t = VI \text{ (Watt)} \times t \text{ (hours)}$$

$$W = \frac{VIt}{1000} \text{ Killowatt-hour (Unit)}$$

So that $\boxed{\text{Number of Units} = \dfrac{\text{Watt} \times \text{hour}}{1000}}$



Q. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more ?

CBSE 2017

Sol. Heat dissipated in a wire is given by

$$H = I^2Rt$$

$$H = I^2 \frac{\rho l}{A} \times t \qquad \left(\because R = \frac{\rho l}{A}\right)$$

Here radius is same, hence area is same. Also current I and length l are same.

$$\therefore \qquad H \propto \rho$$

But $\rho_{nichrome} > \rho_{copper}$

so that $H_{nichrome} > H_{copper}$

CELL

EMF - EMF of a cell is the maximum potential difference between two electrodes of the cell when no current is drawn from the cell. It is denoted by E.

Internal Resistance -

Internal resistance of a cell is defined as the resistance offered by the electrolyte and electrodes of a cell when the electric current flows through it.

Internal Resistance of a cell depends on the following factors -

(i) Distance between the electrodes

(ii) Nature, Concentration and temperature of electrolyte

(iii) The nature of Electrodes.

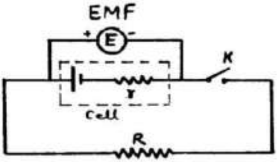
(iv) Area of Electrodes, immersed in the electrolyte

Terminal Potential Difference -

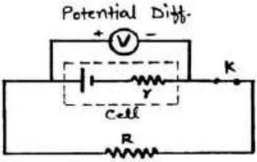
Terminal Potential difference of a cell is defined as the potential difference between the two electrodes of a cell in a closed circuit, when current is drawn from the cell. It is denoted by V.

* The terminal potential difference of a cell is always less than the emf of the cell because of the fall of potential across the internal resistance of the cell.

$$\boxed{V = E - Ir}$$



Open Circuit (EMF is measured)



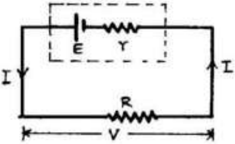
Closed Circuit (P.D. is measured)

According to the definition of terminal potential difference -

$$\boxed{V = E - Ir}$$

Internal Resistance of a Cell in terms of E, V & R

The terminal potential difference of a cell is equal to the potential difference across the external resistance R of the circuit, so -



$$I = \frac{V}{R}$$

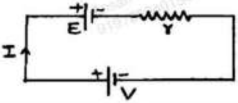
We know that. $Ir = E - V$

So $r = \dfrac{E-V}{I} = \dfrac{E-V}{V/R}$

$$\boxed{r = \left(\frac{E}{V} - 1\right)R}$$

Note - At the time of charging of a cell the terminal potential difference across the cell is

$$\boxed{V = E + Ir}$$



Difference between E.M.F. and Potential difference

EMF	Potential Difference
1. The emf of a cell is the maximum potential difference between the two points when the cell is in the open circuit	1. The potential difference between the two points is the difference of potential between those two points in a closed circuit.
2. It is independent of the resistance of the circuit and depends upon the nature of electrodes and electrolyte of Cell.	2. It depends upon the resistance between the two points of the circuit and current flowing through the circuit.
3. The term emf is used only for the source of emf.	3. It is measured between any two points of the circuit.

Q. CBSE 2016,20 Plot a graph showing the variation of Voltage V with the current drawn from the cell. How can one get information from this plot about the emf of the cell and its internal resistance.

Sol.

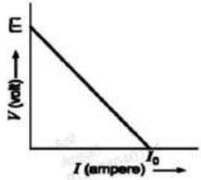
$$V = E - Ir$$

$$r = \frac{E-V}{I}$$

At $I = 0$, $V = E$

when $V = 0$, $I = I_0$

$$r = \frac{E}{I_0}$$



The intercept on y-axis gives the emf of the cell. The slope of graph gives the internal resistance.

Q. (CBSE 2019) A voltmeter of resistance 998 Ω is connected across a cell of emf 2V and internal resistance 2Ω. Find the potential difference across the voltmeter and also across the terminals of the cell. Estimate the percentage error in the reading of the voltmeter.

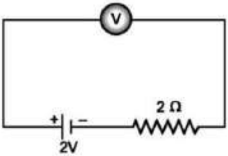
Sol.

$$V = E - Ir$$

$$998 \times I = 2 - 2I$$

$$1000 \times I = 2$$

$$I = 0.002\ A$$



$$V = 0.002 \times 998$$

$$V = 1.996\ V$$

$$\Delta V = 2 - 1.996 = 0.004\ V$$

$$\text{Percentage Error} = \frac{\Delta V}{V} \times 100$$

$$= \frac{0.004}{2} \times 100 = 0.2\%$$

Q. (CBSE 2008) A (a) series (b) Parallel combination of two given resistors is connected, one-by-one across a cell. In which case will the terminal potential difference, across the cell have a higher value?

Sol. We know that equivalent resistance of the combination of resistance is - (a) greater than the greatest resistance in series combination and (b) smaller than the least value of resistance in parallel combination.

The terminal potential difference across the cell is higher in series combination as

$$V = E - Ir$$

and due to higher resistance current I is less in series combination.

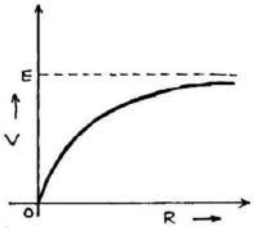
Q. (Delhi 2009) A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing the variation of terminal potential V with resistance R.

Sol. Terminal Voltage V is given by -

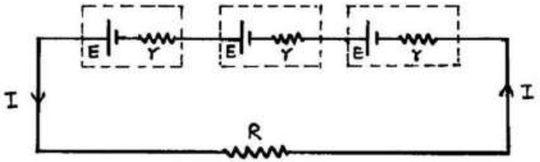
$$V = IR = \frac{E}{(R+r)} \times R$$

$$V = \frac{E}{(1 + r/R)}$$

⇒ so V increases with the increase of R.



Cells in Series Combination



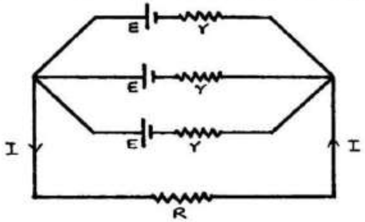
Total emf of the battery = nE (For n cells)

Total internal resistance of the battery = nr

Total resistance of the circuit = $nr + R$

So $\boxed{\text{Current } I = \dfrac{nE}{nr + R}}$

Cells in parallel Combination



Total EMF of the battery = E (For n cells)

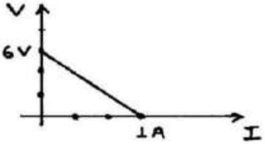
Total internal resistance of the battery = r/n

Total resistance of the circuit = $\dfrac{r}{n} + R$

So $\boxed{\text{Current } I = \dfrac{nE}{R + r/n}}$

Q. The plot of the variation of potential difference across a combination of three identical cells in series versus current is as shown in figure. What is the emf of each cell ?

Delhi 2008



Sol. Terminal Potential difference across the cell combination can be given as-

by $V = E_{eq} - I r_{eq}$

$$V = 3E - I(3r)$$

when $I = 0$ then $V = 3E$

from the graph when $I = 0$, $V = 6$ Volt

$3E = 6$ Volt Hence emf $E = 2$ Volt.

Q. Under what condition will the current in a wire be the same when connected in series and in parallel of n identical cells each having internal resistance r and external resistance R ?

CBSE 2019

Sol. Let n identical cell of internal resistance r connected in series and parallel with external resistance R.

$$I_s = \frac{nE}{R+nr} \quad \text{and} \quad I_p = \frac{E}{R+\frac{r}{n}} = \frac{nE}{Rn+r}$$

According to question $I_s = I_p$

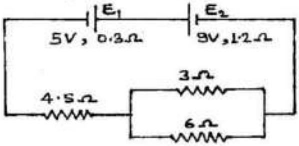
$$\frac{nE}{R+nr} = \frac{nE}{Rn+r}$$

$$R+nr = Rn+r$$

$$nr - r = Rn - R$$
$$r(n-1) = R(n-1)$$
$$r = R$$

So the current will be same, when internal resistance of cell r is equal to external resistance R.

Q. (Delhi 2006) Two cells E_1 and E_2 in the given circuit diagram have an emf of 5 V and 9 V and internal resistance of $0.3\ \Omega$ and $1.2\ \Omega$ respectively. Calculate the value of current flowing through the resistance of $3\ \Omega$.



Sol.

$$\text{Net emf} = 9 - 5 = 4\ \text{V}$$

Equivalent resistance of $6\ \Omega$ and $3\ \Omega$ is

$$= \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Net resistance across the circuit

$$R = 0.3 + 1.2 + 2 + 4.5$$
$$R = 8\ \Omega$$

So current $I = \frac{V}{R} = \frac{4}{8} = 0.5\ \text{A}$

Potential difference across $(4.5 + 1.5)\ \Omega$ is

$$V = IR$$
$$V = 0.5 \times 6 = 3\ \text{Volt}$$

$\therefore$ Potential difference across $6\ \Omega$ and $3\ \Omega$

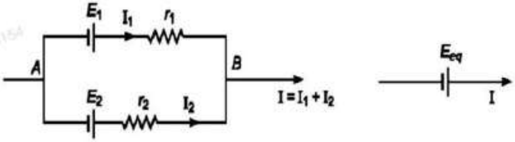
$$= 4 - 3 = 1\ \text{Volt}$$

$\therefore$ Current through resistance $3\,\Omega$

$$I' = \frac{V}{R} = \frac{1}{3} = 0.33\ A$$

Q. (CBSE 2015) Two cells of emf E_1 and E_2 and internal resistance r_1 and r_2 are connected in parallel. Derive the expression for the (i) EMF and (ii) internal resistance of equivalent cell.

Sol.



Let I_1 and I_2 be the currents leaving the positive terminals of the cells, and at the point B

$$I = I_1 + I_2 \quad \text{— (1)}$$

Let V be the potential difference between points A and B of the combination, so

$$V = E_1 - I_1 r_1 \quad \text{— (2) [across the cells]}$$

and

$$V = E_2 - I_2 r_2 \quad \text{— (3)}$$

from equation (1), (2) and (3) we get

$$I = \frac{(E_1 - V)}{r_1} + \frac{(E_2 - V)}{r_2}$$

$$\left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{— (4)}$$

For the equivalent cell,

$$V = E_{eq} - I r_{eq}$$

or

$$I = \frac{E_{eq}}{r_{eq}} - \frac{V}{r_{eq}} \quad \text{— (5)}$$

On Comparing the equations (4) and (5) we get

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

and $$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \Rightarrow E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

Q. (CBSE 2016) Two cells of emfs 1.5 V and 2 V having internal resistance 0.2 Ω and 0.3 Ω respectively are connected in parallel. Calculate the emf and internal resistance of the equivalent cell.

Sol. Given that $E_1 = 1.5\,V$, $r_1 = 0.2\,\Omega$
$E_2 = 2\,V$, $r_2 = 0.3\,\Omega$

EMF of equivalent cell

$$E = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$E = \frac{1.5 \times 0.3 + 2 \times 0.2}{0.2 + 0.3} = 1.7\,V$$

Internal resistance of equivalent cell

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} \Rightarrow r = \frac{r_1 r_2}{r_1 + r_2}$$

$$r = \frac{0.2 \times 0.3}{0.2 + 0.3} = 0.12\,\Omega$$

Q. CBSE 2007 A cylinderical metallic wire is stretched to increase its length by 10%. Calculate the percentage change in its resistance.

Sol. Volume of the wire $V = AL$

$$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{\Delta A}{A} \times 100 + \frac{\Delta L}{L} \times 100$$

But $\Delta V = 0$ (Volume remains Constant)

$$\Rightarrow \frac{\Delta A}{A} \times 100 = -\frac{\Delta l}{l} \times 100$$

Negative sign imply that increase in length of conductor would create decrease in area.

$$\because \quad R = \rho L/A$$

For given resistor ρ = constant

So $$\frac{\Delta R}{R} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) - \left(\frac{\Delta A}{A} \times 100\right)$$

Here $\frac{\Delta l}{l} \times 100 = 10\%$

Therefore $\frac{\Delta A}{A} \times 100 = -10\%$

$$\therefore \quad \frac{\Delta R}{R} \times 100 = 10\% - (-10\%)$$

$$\frac{\Delta R}{R} \times 100 = 20\%$$

$\therefore$ Percentage Rise of resistance = 20 %.

Kirchhoff's Law

(1) Kirchhoff's Current Law (KCL)

The algebric sum of the currents meeting at a junction in a closed electric circuit is zero.

$$I_{exiting} + I_{entering} = 0$$
$$\sum I = 0$$

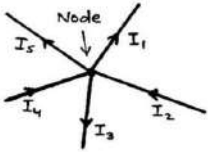
Sign Convention-

(1) The incoming currents towards the junction are taken positive.

(2) The outgoing currents away from the junction are taken negative.

By Kirchhoff's Current Law-

$$I_2 + I_4 - I_1 - I_3 - I_5 = 0$$



* Kirchoff's current Law supports law of **conservation of charge.**

The charges cannot accumulate at a junction. The number of charges that arrive at a junction in a given time must leave in the same time in accordance with conservation of charges.

(2) Kirchhoff's Voltage Law (KVL)

The algebric sum of all the potential drops and emf's along any closed path in an electrical network is always zero.

$$\boxed{\Sigma \Delta V = 0}$$

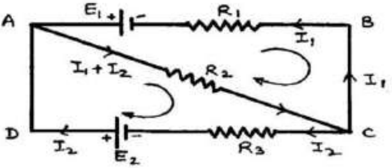
Sign Convention-

(1) The emf is taken negative when we traverse from positive to negative terminal of the cell.

(2) The emf is taken positive when we traverse from negative to positive terminal of the cell.

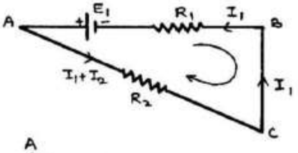
(3) If the resistor is being traversed in the direction of the current then potential difference across it, is taken negative i.e. $-IR$.

(4) If the resistor is being traversed in the direction opposite to the current then potential difference across it, is taken positive i.e. IR.



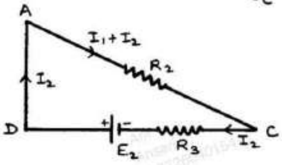
For Loop ABCA

$$\boxed{-E_1 + I_1R_1 + (I_1+I_2)R_2 = 0}$$



For Loop ACDA

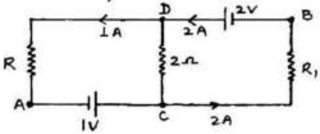
$$\boxed{-(I_1+I_2)R_2 - I_2R_3 + E_2 = 0}$$



* Kirchoff's Voltage Law (KVL) supports the Law of conservation of energy.

Q. All India 2011

In the given circuit, assuming point A to be at zero potential, use Kirchhoff's Laws to determine the potential at point B.



Sol.

By KCL at point D

$$I_{DC} + 1 = 2$$

$$I_{DC} = 1\ A$$

Along the ACDBA

$$V_B - V_A = +1 + 2\times1 - 2 = +1$$

But $V_A = 0$, so that

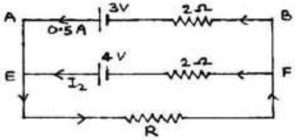
$$V_B = 1\ V$$

Q. All India 2011

Using Kirchhoff's rules in the given circuit, find

(a) the voltage drop across the unknown resistor R.

(b) the current I_2 in the arm EF.



Sol.

Applying KVL in the closed loop ABFEA

$$-3 + 1 - 2I_2 + 4 = 0$$

$$I_2 = 1\ A$$

So the current I_2 in the arm EF is 1A.

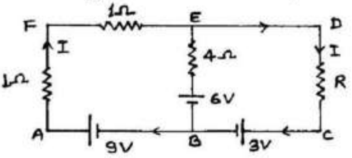
Potential drop across R, EF and AB is same because these are parallel.

So that $\quad V = V_F - V_E = -4 + 2\times1$

$$V = 2\ \text{Volt}.$$

Q. Delhi 2012

Using Kirchoff's rules, determine the value of unknown resistance R in the circuit, so that no current flows through 4Ω resistance. Also find the potential difference between points A and D.



Sol. Given that no current should flow through 4Ω resistance, So Let Current I flows through the Loop AFEDCBA.

By applying KVL in loop AFEBA -

$$-1\times I - 1\times I - 6 + 9 = 0$$

So $$I = \frac{3}{2}\ A$$

Now applying KVL in loop AFDCA -

$$-1\times\frac{3}{2} - 1\times\frac{3}{2} - R\times\frac{3}{2} - 3 + 9 = 0$$

$$R\times\frac{3}{2} = 3$$

So $$R = 2\ \Omega$$

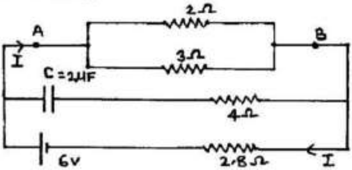
For potential difference across points A and D, along AFED -

$$V_D - V_A = -1\times\frac{3}{2} - 1\times\frac{3}{2}$$

$$= -3\ V$$

Q. Calculate the steady current through the 2Ω resistor in the given circuit

CBSE 2010



Sol. No current flows through 4Ω resistor in steady state as Capacitor offers infinite resistance in DC Circuit.

Here 2Ω and 3Ω are in parallel Combination

So $$R_{AB} = \frac{2 \times 3}{2+3} = 1.2 \text{ A}$$

So Equivalent Resistance of the circuit is

$$R_{eq} = 1.2 + 2.8 = 4\,\Omega$$

so total Current $$I = \frac{6}{4} = 1.5 \text{ A}$$

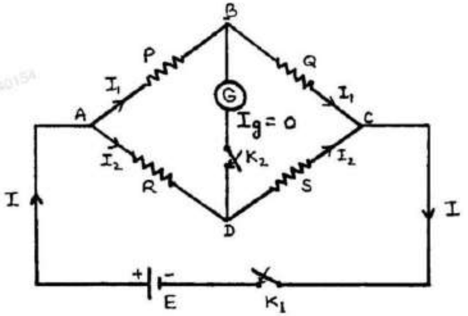
and Current through 2Ω resistor is given by -

$$I' = 1.5 \times \frac{3}{(2+3)} \Rightarrow I' = 0.9 \text{ A}$$

Wheatstone Bridge

It is an arrangement of four resistors used to determine resistance of one resistors in terms of other three resistors.

Consider the figure given below, which is an arrangement of resistors and is known as wheatstone bridge.



Wheatstone bridge consist of four resistance P, Q, R and S, with a battery of emf E.

On pressing key K_2 if galvanometer does not show any deflection then wheatstone bridge is said to be balanced.

Galvanometer is not showing any deflection this means that no current is flowing through the galvanometer and terminal B and D are at the same potential.

Thus for a balanced bridge $\boxed{V_B = V_D}$

Now we have to find the condition for the balanced wheatstone bridge. For this by applying KVL to the loop ADBA -

$$-I_2 R + I_1 P = 0$$

or $$I_1 P = I_2 R \qquad \text{—} ①$$

Again by applying KVL to the loop BCDB -

$$-I_1 Q + I_2 S = 0$$

or $$I_1 Q = I_2 S \qquad \text{—} ②$$

from equation ① and ② we get -

$$\frac{I_1}{I_2} = \frac{R}{P} = \frac{S}{Q}$$

or $$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

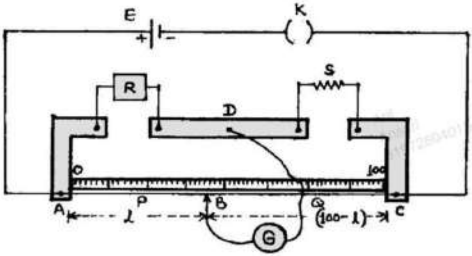
This equation gives the Condition for the balanced wheatstone bridge.

Thus if the ratio of the P and Q and the resistance R is known then unknown resistance S can easily be Calculated.

Meter Bridge

A slide wire bridge is a practical form of Wheatstone bridge.

It consist of a wire AC of constantan or manganin of 1 meter length and of uniform area of cross-section.



The wire is stretched between the Copper strips on a horizontal wooden board. A meter scale is also fitted on the wooden board parallel to the length of the wire.

There is another copper strip fitted on the wooden board in order to provide two gaps in strips. Across one gap, a resistance box R and in another gap the unknown resistance S are Connected.

A jokey B can be slided on the bridge wire. One terminal of the galvanometer G is Connected to jokey B and the other to the terminal D.

The positive pole of the battery E is Connected to terminal A and the negative pole of the battery to terminal C through Key K.

Working - Close Key K and take out suitable resistance R from resistance box. Adjust the position of jockey on the wire (say at B) where on pressing, the galvanometer shows no deflection. Note the length AB ($= l$) of the wire. Find the length BC ($= 100 - l$) of the wire.

Now as the bridge is balanced, therefore, according to wheatstone bridge principle-

$$\frac{P}{Q} = \frac{R}{S}$$

If r is the resistance per centimeter length of wire then -

P = resistance of the length l of wire AB $= lr$

Q = resistance of the length $(100 - l)$ of the wire BC $= (100 - l)r$

$$\therefore \quad \frac{lr}{(100 - l)r} = \frac{R}{S}$$

or

$$\boxed{S = \left(\frac{100 - l}{l}\right) \times R}$$

So by knowing l and R we can calculate S.

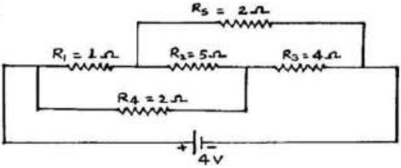
* Characteristics of mangnin wire which make it suitable for making a standard resistance -

(i) Low temperature coefficient of resistance

(ii) High value of resistivity of material of Manganin.

Q. Calculate the current drawn from the battery in the given network.

All India 2009

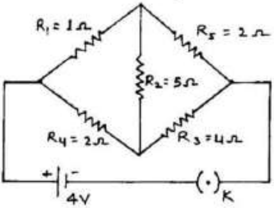


Sol. The given circuit can be redrawn as given below.

Here $\frac{R_1}{R_5} = \frac{R_4}{R_3}$

$\Rightarrow \frac{1}{2} = \frac{2}{4}$

wheatstone bridge is balanced so there will be no current in the resistance R_2 and it can be withdrawn.



The equivalent resistance would be equivalent to a parallel combination of two rows which consist of series combination of R_1 and R_5, and R_4 and R_3.

$$\frac{1}{R} = \frac{1}{1+2} + \frac{1}{2+4} = \frac{1}{2}$$

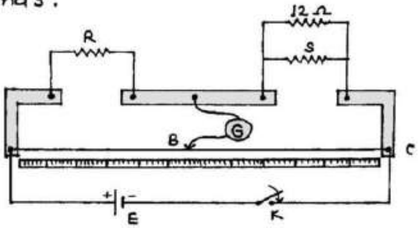
$$R = 2\Omega$$

$$\therefore \quad I = \frac{V}{R} = \frac{4}{2} = 2\,A$$

$$I = 2\,A$$

Q. (Delhi 2010) In a meter bridge the null point is found at a distance of 40 cm from A. If a resistance of 12 Ω is connected in parallel with S, the null point occurs at 50 cm from A. Determine the value of R and S.

Sol.



In the null point condition the bridge is balanced. So by applying the condition of balanced wheatstone bridge,

$$\frac{R}{S} = \frac{l}{100-l} = \frac{40}{100-40} = \frac{2}{3} \Rightarrow \frac{R}{S} = \frac{2}{3}$$

The equivalent resistance of 12 Ω and S in parallel is

$$= \frac{12\,S}{12+S}\ \Omega$$

Again applying the condition

$$\frac{R}{\left(\frac{12\,S}{12+S}\right)} = \frac{50}{50} = 1$$

$$R = \frac{12\,S}{12+S}$$

or $$\frac{2}{3}S = \frac{12\,S}{12+S} \qquad \left[\text{as } R = \frac{2}{3}S\right]$$

$$12 + S = 18$$

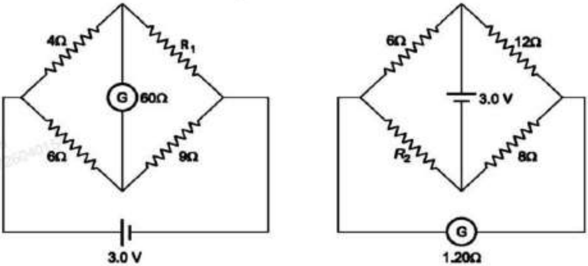
$$S = 6\ \Omega$$

$$R = \frac{2}{3}S = \frac{2}{3}\times 6$$

So $$R = 4\ \Omega$$

Q. (CBSE 2013) Fig shows two circuits each having a galvenometer and a battery of 3V. When the galvenometer in each arrangement do not show any deflection, obtain the ratio $\frac{R_1}{R_2}$.

Sol.



For balanced wheatstone bridge, if no current flows through the galvenometer, then

$$\frac{4}{R_1} = \frac{6}{9} \Rightarrow R_1 = \frac{4\times 9}{6} = 6\,\Omega$$

For another circuit

$$\frac{6}{12} = \frac{R_2}{8} \Rightarrow R_2 = \frac{6\times 8}{12} = 4\,\Omega$$

$$\therefore \frac{R_1}{R_2} = \frac{6}{4} \Rightarrow \frac{R_1}{R_2} = \frac{3}{2}$$

YOUR REAL STRENGTH COMES FROM
BEING THE BEST "YOU" YOU CAN BE.

Potentiometer

Potentiometer is an accurate instrument used to compare emf' of cells, to measure the emf of a cell or potential difference between two points in an electrical circuit accurately.

Principle-

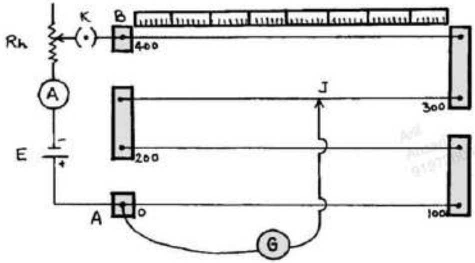
potentiometer is based on the principle that potential drop across any portion of the wire of uniform cross-section is proportional to the length of that portion of the wire when a constant current flows through the wire.

Construction & Working

A potentiometer consist of a long uniform wire of manganin or constantan, stretched on a wooden board. A meter scale is fixed on the board parallel to the length of the wire.

The potentiometer is provided with a jockey J with the help of which, the contact can be made at any point of the wire.

A battery E is connected across A and B, which sends the current through the wire which is kept constant by using a rheostat R_h.
This circuit is called an auxillary circuit.



Suppose A and ρ are the area of cross-section and resistivity of the material of the wire, then

Resistance $R = \frac{\rho l}{A}$

where l = length of the wire.

If Current I flowing through the wire then by ohm's Law

$$V = IR$$

where V is the potential difference across the wire of length l, thus -

$$V = IR = \frac{I\rho l}{A} = Kl$$

$\therefore$ $\boxed{V \propto l}$ (where $K = \frac{I\rho}{A}$)

Hence potential difference across any portion of potentiometer wire is directly proportional to length of the wire of that portion.

* Here $K = \frac{V}{l}$ is called potential gradient i.e. the fall of potential per unit length of the wire.

Dependancy of Potential Gradient

by formula $K = \frac{E_s}{l} = \frac{I\rho}{A} = \frac{I\rho}{\pi r^2}$

(i) On current ($K \propto I$)

(ii) On Resistivity ($K \propto \rho$)

(iii) On the radius of wire ($K \propto \frac{1}{r^2}$)

Sensitivity of a Potentiometer -

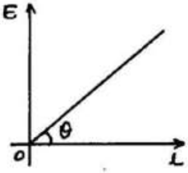
Sensitivity of a potentiometer depends on the potential gradient (K). Lesser the potential gradient, higher will be the sensitivity of the potentiometer.

$$\boxed{\text{Sensitivity} \propto \frac{1}{\text{Potential Gradient}}}$$

If the potential gradient is less then the balance length on the potentiometer wire is increase and the length can be measured with more accuracy.

It is the reason due to which a long resistance wire is used in potentiometer.

by graph between the applied potential difference and balance length l -

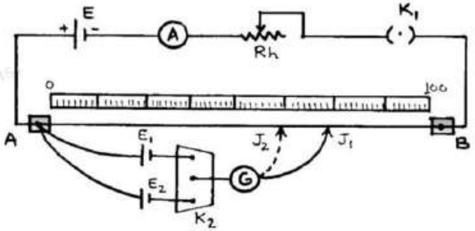


Slope of the graph = $\tan\theta = \frac{E}{l}$

$= K$ (Potential Gradient)

(A) Comparison of EMF's of two cells using potentiometer -

Positive terminals of two cells of emf's E_1 and E_2 (whose emf are to be compared) are connected to the terminal A and negative terminals are connected to jockey through a two way Key K_2 and a galvanometer.



⇨ Now key K_1 is closed to establish a potential difference between the terminals A and B then by closing Key K_2 introduce cell of emf E_1 in the circuit.

Now determine the null point J_1 with the help of jockey. If the null point on the wire is at length l_1 from A then - $E_1 = Kl_1$

where K is the potential gradient of wire.

⇨ Similarly cell of emf E_2 is introduced in the circuit and again null point J_2 is determined. If length of this null point from A is l_2 then

$$E_2 = Kl_2$$

Therefore, we get $\boxed{\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}}$

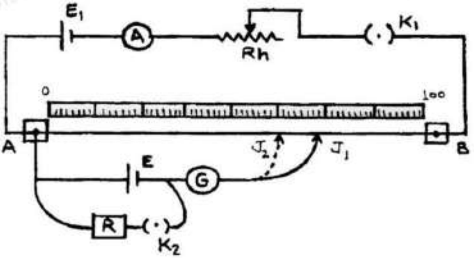
Hence we can find the ratio E_1 / E_2.

* If the emf of one cell is Known then the emf of other cell can be Known easily.

* A balance point can not be obtained on the wire if the fall of potential along the potentiometer wire due to driving cell is less than the emf of the emf of the cell to be balanced.

(B) Determination of internal resistance of the cell

The cell whose internal resistance is to be determined is connected to the terminal A of the potentiometer across a resistance box through a key K_2.



⇨ First close the key K_1 and obtain the null point. Let l_1 be the length of this null point from terminal A then -

$$E = K l_1 \qquad (\text{where } l_1 = AJ_1)$$

⇨ when key K_2 is closed, the cell sends current through resistance box (R).

If V is the terminal potential difference and null point is obtained at length l_2, then -

$$V = K l_2 \qquad (\text{where } l_2 = AJ_2)$$

Thus $$\boxed{\frac{E}{V} = \frac{l_1}{l_2}}$$

We Know that the internal resistance r of a cell of emf E, when a resistance R is connected in its circuit is given by -

$$r = \left(\frac{E}{V} - 1\right) R$$

$$\boxed{r = \left(\frac{l_1}{l_2} - 1\right) R}$$

Thus by Knowing the value of l_1, l_2 and R, the internal resistance r of the cell can be determined.

Difference between voltmeter and potentiometer

S.N	Voltmeter	Potentiometer
1.	Its resistance is high but finite.	Its resistance is infinite
2.	It draws some current from Source of emf.	It does not draw any current from the source of emf.
3.	Its sensitivity is low	Its sensitivity is high.
4.	It is based on deflection method.	It is based on null deflection method.
5.	The potential difference measured by it is lesser than the actual value.	The potential difference measured by it is equal to actual value.

Q. The current in the primary circuit of a potentiometer is 0.2A. If the cross section area of the wire is $0.8 \times 10^{-6} m^2$ and its resistivity is $40 \times 10^{-8} \Omega m$ then find the potential gradient.

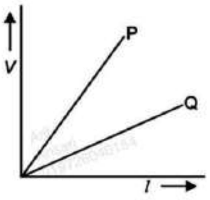
<u>Answer</u>: 0.1 V/m

Q. The variation of potential difference V with length l in the case of two potentiometer P and Q is shown in fig. Which of these two will you prefer for comparing the emf of two primary cells and why?

CBSE 2016, 19

<u>Sol.</u> For greater sensitivity or accuracy of potentiometer, the potential gradient $\frac{V}{l}$ (slope of V-l graph) should be small.

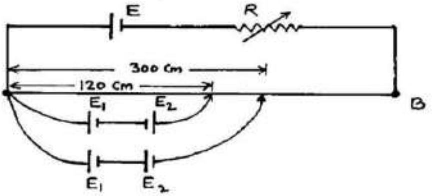
In the given graph the slope $\frac{V}{l}$ is smaller for a potentiometer Q, hence we shall prefer potentiometer Q, for comparing the emfs of two cells.



Q. In a potentiometer the null point for the two primary cells of emf E_1 and E_2 connected in the given manner, are obtained at a distance of 120 cm. and 300 cm. from the end A. Find (a) E_1/E_2 and (b) position of null point for the cell E_1.

Delhi 2012

How is the sensitivity of a potentiometer increased?



Sol. (a) Let the potential gradient be K.

$\because$ $E_1 - E_2 = K \times 120$ (Cells are connected in opposite manner)

and $E_1 + E_2 = K \times 300$ (cells are connected in supporting manner)

so that $\dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{K \times 300}{K \times 120} = \dfrac{5}{2}$

Now apply Componendo and dividendo -

$$\frac{(E_1 + E_2) + (E_1 - E_2)}{(E_1 + E_2) - (E_1 - E_2)} = \frac{5+2}{5-2}$$

so $\dfrac{E_1}{E_2} = \dfrac{7}{3}$

(b) $E_2 = \dfrac{3}{7} E_1$

or $E_1 - \dfrac{3}{7} E_1 = K \times 120$

$\dfrac{4}{7} E_1 = K \times 120$ or $E_1 = K \times 210$

Hence null point for E_1 is obtained at 210 cm.

* The sensitivity of potentiometer can be increased by increasing the length of wire.

Q.1. In a Hydrogen atom, electron moves in an orbit of radius 5×10^{-11} m with a speed of 2.2×10^6 m/s. Calculate the equivalent current.

Solution: Current $i = f.e = \dfrac{v}{2\pi r}.e$

$$= \frac{2.2\times10^{6}}{2\pi\times5\times10^{-11}} \times 1.6 \times 10^{-19}$$

$$= 1.12 \times 10^{-3} \text{ amp} = 1.12 \text{ mA.}$$

Q.2.* The current through a wire depends on time as $i = i_0 + \alpha t$, where $i_0 = 10A$ and $\alpha = 4A/s$. *Find the charge that crossed through a section of the wire in 10 seconds.

Solution:

$$i = i_0 + \alpha t \text{ ; but } i = \frac{dq}{dt}$$

$$\Rightarrow dq = (i_0 + \alpha t)dt$$

$$q = \int_{t=0}^{t=10} dq \Rightarrow q = \left[i_0 t + \frac{\alpha t^2}{2} \right]_0^{10}$$

$$= (10 i_0 + 50\alpha) = 300 \text{ coulomb}$$

Q.3. Consider a wire of length 4m and cross-sectional area 1 mm² carrying a current of 2A. If each cubic meter of the material contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

Solution:

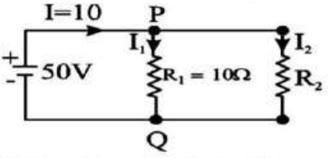
$$\Rightarrow v_d = \frac{i}{nAe} = \frac{2}{10^{29}\times10^{-6}\times1.6\times10^{-19}} ms^{-1}$$

$$= 12.5 \times 10^{-4} \text{ ms}^{-1}$$

Average time taken by an electron to cross the length of wire

$$t = \frac{l}{v_d} = \frac{4}{1.25\times10^{-4}} \text{ s} = 3.2 \times 10^4 \text{ s}$$

Q.4. *For a circuit shown in fig. Find the value of resistance R_2 and current I_2 flowing through R_2*



Solution :

If equivalent resistance of parallel combination of R_1 and R_2 is R, then

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10R_2}{10 + R_2}$$

According to Ohm's law, $R = \frac{V}{I}$

$$R = \frac{50}{10} = 5\Omega \Rightarrow \frac{10R_2}{10 + R_2} = 5 \Rightarrow R_2 = 10\Omega.$$

The current is equally divided into R_1 and R_2. Hence $I_2 = 5A$.

Q.5. *A lamp of 100W works at 220 volts. What is its resistance and current capacity ?*

Solution :

Power of the lamp, $P = 100W$

Operating voltage, $V = 220V$

Current capacity of the lamp,

$$i = \frac{P}{V} = \frac{100}{220} = 0.455A$$

Resistance of the lamp, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

Q.6. *A 100W – 220V bulb is connected to 110V source. Calculate the power consumed by the bulb.*

Solution :

Power of the bulb, $P = 100W$

Operating voltage, $V = 200V$

Resistance of the bulb, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

Actual operating voltage, $V^1 = 110\text{ V}$

Therefore, power consumed by the bulb,

$$P^1 = \frac{(V^1)^2}{R} = \frac{(110)^2}{484} = 25\text{W}.$$

Q.7. When a current drawn from a battery is 0.5A, its terminal potential difference is 20V and when current drawn from it is 2.0A, the terminal voltage reduces to 16 V. Find out the e.m.f and internal resistance of the battery.

Solution: We know

$V = E - Ir$; $I = 0.5\text{ A}, V = 20$ Volt, we have

$20 = E - 0.5r$ .... (i)

$I = 2\text{ A}, V = 16$ Volt, we have

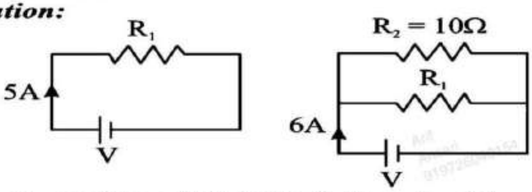
$16 = E - 2r$ .... (ii)

By solving (i) and (ii)

we get $E = 21.3\text{V}, r = 2.675\,\Omega$.

Q.8. An ideal battery passes a current of 5A through a resistor. When it is connected to another resistor of resistance of 10Ω ***in parallel, the current is 6A. Find the resistance of the first resistor.***

Solution:



Current through R_1 in the first case $i_1 = 5\text{A}$

Current in the second case $i_2 = 6\text{A}$

Effective resistance in the second case

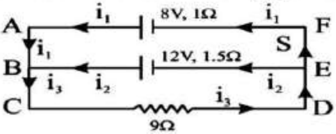
$$R = \frac{R_1R_2}{R_1+R_2}\,;\; V = I_1R_1 \text{ and } V = I_2\frac{R_1R_2}{R_1+R_2}$$

$$I_1R_1 = I_2\frac{R_1R_2}{R_1+R_2} \qquad \Rightarrow I_1 = I_2\frac{R_2}{R_1+R_2}$$

$$5 = 6\times\frac{10}{R_1+10} \qquad \Rightarrow 5(R_1+10) = 60$$

$$5R_1 + 50 = 60,\; 5R_1 = 10 \qquad \Rightarrow R_1 = \frac{10}{5} = 2\,\Omega$$

Q.9. Solve for current values in figure.



Solution :

Applying Kirchhoff's first law at the junction B we have $\quad i_1 + i_2 = i_3 \qquad ....\ (1)$

Applying Kirchhoff's second law to loop ABEFA

$-12 + i_2 \times 1.5 - i_1 \times 1 + 8 = 0$

$\quad i_1 - 1.5\, i_2 = -4. \qquad ....\ (2)$

From loop BCDEB

$-(i_2 \times 1.5) - (i_3 \times 9) + 12 = 0$

$\quad 1.5\, i_2 + 9i_3 = 12 \qquad ....\ (3)$

on solving $i_1 = -1A \quad$ and $i_3 = 1\,A$

Q.10. The length of a potentiometer wire is 1m and its resistance is 4Ω. A current of 5 mA is flowing in it. An unknown source of e.m.f is balanced on 40 cm length of this wire, then find the e.m.f of the source.

Solution:

$$x = I\rho = I\frac{R}{L} = \frac{5\times 4}{1} = 20\,mV$$

$$E = l\,x = 0.\,40 \times 20 = 8\text{ mV}$$

Q.11. A cell of e.m.f 2 volt and internal resistance* 1.5Ω *is connected to the ends of 1m long wire. The resistance of wire is* $0.5\Omega/m$. *Find the value of potential gradient on the wire.

Solution:

$$X=\frac{IR}{L}=\left(\frac{E}{R+r}\right)\frac{R}{L}=\frac{2\times0.5}{0.5+1.5}=\mathbf{0.5\ V/m}$$

Q.12. In a potentiometer experiment, when a battery of e.m.f. 2V is included in the secondary circuit, the balance point is 500cm. Find the balancing length of the same end when a cadmium cell of e.m.f. 1.018V is connected to the secondary circuit.

Solution:

$$E \propto \ell$$

$$\frac{E_1}{E_2}=\frac{\ell_1}{\ell_2}$$

$$\ell_2=\frac{E_2}{E_1}\times\ell_1=\frac{1.018}{2}\times500=254.5\,\text{cm}.$$